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OPTIMUM SUCTION DISTRIBUTION TO OBTAIN A LAMINAR BOUNDARY LAYER

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Аннотация—Излагается метод расчёта ламинарного пограничного слоя с оптимальным распределением отсоса.

Использование в качестве первого приближения зависимости $H(\bar{x})$ для пластины с оптимальным остосом позволило построить исчерпывающие графики (рис. 2–4), значительно упрощающие расчёт трения и критического числа Re_{kp}^{**} по длине тела.

Определение оптмиальной скорости отсоса и параметра H ироизводится соответственно по формуле (11) и формуле (16).

NOMENCLATURE

u_o, approach flow velocity at infinity;

u, stream velocity at outer boundary layer;

v_o, suction velocity;

L, length of body of revolution;

$r_o(x)$, equation of meridional section of body of revolution;

r, distance from body axis to arbitrary point in boundary layer;

U, streamwise velocity component in boundary layer at distance y from surface;

δ^*, attenuation thickness;

δ^{**}, momentum thickness;

δ^{***}, energy loss thickness;

$$\bar{u} = \frac{u}{u_o}; \quad r_o = \frac{r_o}{L}; \quad \bar{x} = \frac{x}{L};$$

$$Re = \frac{uL}{\nu};$$

$$Re^{**} = \frac{u\delta^{**}}{\nu};$$

$$H = \frac{\delta^*}{\delta^{**}};$$

$$Re_{\bar{x}} = \frac{x}{L}\, Re;$$

$$\mathscr{D} = \int_0^\infty \frac{r}{r_o}\left[\frac{\partial\,(U/u)}{\partial\,(y/\delta^{**}}\right]^2 \mathrm{d}\left(\frac{y}{\delta^{**}}\right)$$

Re_{I}^{**}, value of Re^{**} in the point of stability loss.

1.

Suction of liquid from a boundary layer through a porous surface (distributed suction) or through the slots distributed over a surface (discontinuously distributed suction) is an effective mean for increasing boundary-layer stability and preventing separation.

Practical applications of available calculation methods for boundary-layer parameters with distributed suction are laborious.

2.

Momentum and energy equations for a boundary layer with distributed suction are similar to those for a boundary layer on an impenetrable surface of a body of revolution, assuming the normal velocity component on the wall is not zero

$$\frac{\mathrm{d}\delta^{**}}{\mathrm{d}x} + \frac{\mathrm{d}u}{\mathrm{d}x}\,\frac{\delta^{**}}{u}\left(2 + \frac{\delta^*}{\delta^{**}}\right) + \frac{\mathrm{d}r_o}{\mathrm{d}x}\,\frac{\delta^{**}}{r_o} - \frac{v_o}{u} = \frac{\tau_o}{\rho u^2}, \tag{1}$$

$$\frac{1}{u^3 r_o}\,\frac{\mathrm{d}}{\mathrm{d}x}(u^3 r_o \delta^{***}) - \frac{v_o}{u} = \frac{2\mathscr{D}}{Re^{**}}. \tag{2}$$

The term v_o/u in (1) and (2) describes the momentum change due to suction on the wall.

With distributed suction rigorous solutions of the boundary-layer equations are possible only for a few specific problems. For an approximate solution the true velocity distribution in a boundary layer is replaced by an approximate one, which simplifies the differential equations of the boundary layer.

3.

From equations (1) and (2) according to Wieghardt [1] two differential equations in terms of the parameter $H = \delta^*/\delta^{**}$ and $Re^{**} = \delta^{**}u/\nu$ are derived.

To calculate the optimum law of the suction velocity distribution along the body, a condition of stability is used in the form of a relation between Re^{**}_{cr} and the parameter H

$$Re^{**}_{cr} = \exp(a - bH) \tag{3}$$

where $a = 26{\cdot}3$; $b = 8$.

The stability parameter chosen over the range of interest $2{\cdot}0 \leqslant H \leqslant 2{\cdot}6$ yields a somewhat underestimated value for the point of stability loss compared with the relations reported by other authors, Wieghardt included, and these are on the safe side.

The value of critical Re^{**} for an asymptotic suction profile

$$\frac{U}{u} = 1 - \exp\frac{v_o y}{\nu} \tag{4}$$

agrees well with the law of $Re^{**}_{cr}(H)$ chosen.

The derived system of three equations is solved with respect to Re^{**} and the optimum suction velocity v_o/u_o

$$Re^{**2} - Re_1^{**2} = 16f_3\, Re\, \bar{u}^{-16f_1}\, \bar{r}_o^{-16f_2} \int_{\bar{x}_1}^{\bar{x}} \bar{u}^{1-16f_1}\, \bar{r}_o^{16f_2}\, \mathrm{d}x, \tag{5}$$

$$\frac{v_o}{u_o} = \frac{Re^{**}}{Re}\left(C_1\frac{\mathrm{d}\bar{u}}{\bar{u}\,\mathrm{d}x} + C_2\frac{\mathrm{d}\bar{r}_o}{\bar{r}_o\,\mathrm{d}\bar{x}}\right) + C_3\frac{\bar{u}}{Re^{**}}. \tag{6}$$

$16f_1$, $16f_2$, $16f_3$ and C_1, C_2, C_3 are some subsidiary functions calculated from the data of [2] for velocity profiles proposed by the author. These profiles are more exact approximations of the true velocity distribution in a boundary layer than the Schlichting profiles which are used by Wieghardt [1].

The function $f_2(H)$ over the range $2{\cdot}0 \leqslant H \leqslant 2{\cdot}6$ may be assumed constant $f_2 = 0{\cdot}125$. Hence $16f_2 = 2$, which agrees with the value of the

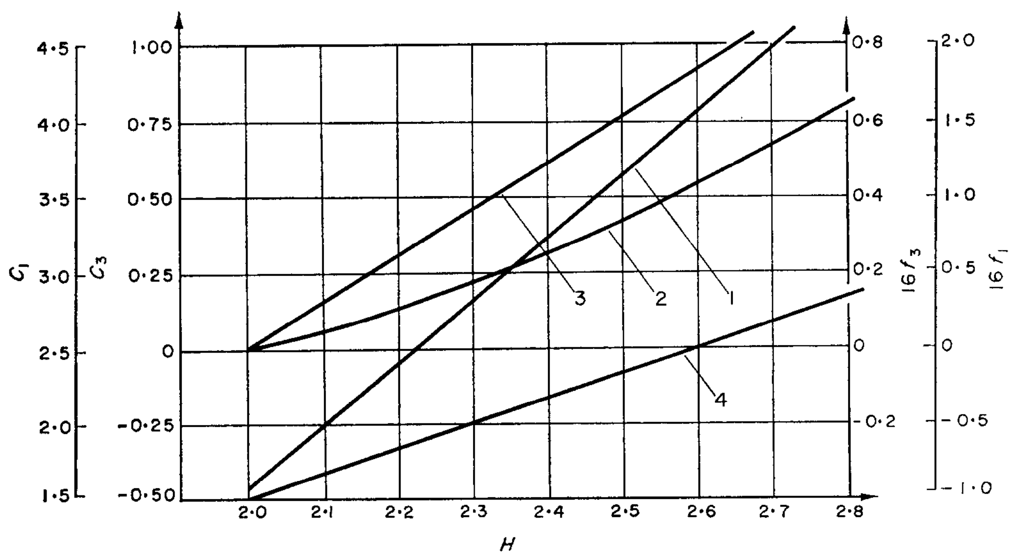

FIG. 1. Variation of subsidiary functions $16f_1$, $16f_3$, C_1 and C_3 with parameter H, 1. $16f_1(H)$. 2. $16f_3(H)$. 3. $C_1(H)$. 4. $C_3(H)$.

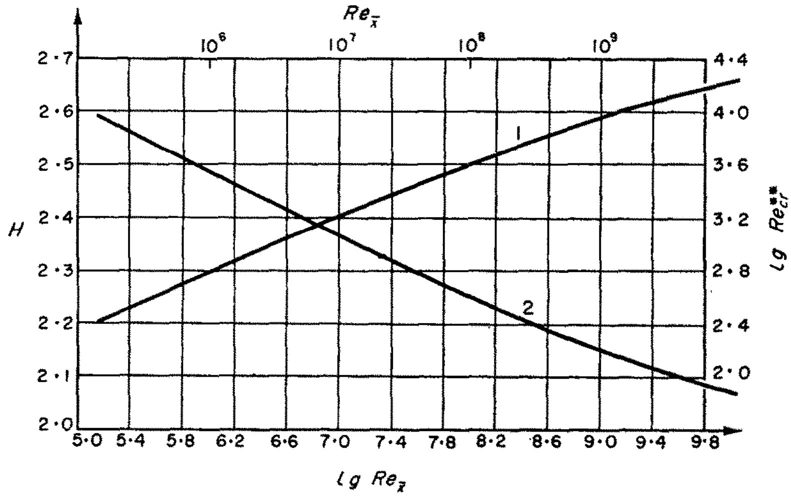

FIG. 2. Variation of Re_{cr}^{**} and parameter H with local $Re_{\bar{x}}$ for plate with optimum suction 1. $Re_{cr}^{**}(Re_{\bar{x}})$ 2. $H(Re_{\bar{x}})$.

exponent of $\bar{r}_0$ for calculation of a boundary layer on impenetrable bodies. Thus in equation (6), $C_2 = 0$.

The relations $16f_1(H)$, $16f_3(H)$ and $C_1(H)$, $C_3(H)$ are plotted in Fig. 1.

4.

Since $H = f(Re^{**})$, estimation of the boundary-layer properties according to formula (5) is possible only by several approximations, similar to those proposed by Truchenbrodt [3] in his method, in which only the integral relation for a boundary layer is used.

The method proposed by Wieghardt [1] is as cumbersome as the previous one. In this method from the system of three equations an equation for H is obtained from which the calculation formula is derived by linearizing for a small increment, which requires increase of the number of calculation points.

In the present paper $H(x)$ is proposed as a first approximation for a plate with the optimum suction.

From equation (5) for the case of a flat plate we have

$$Re_{\bar{x}} - Re_{\bar{x}_1} = \frac{Re^{**2} - Re_1^{**2}}{16f_3}. \qquad (7)$$

Using $Re^{**}(H)$ and $16f_3(H)$, we obtain the relation $Re_{\bar{x}} - Re_{\bar{x}_1} = f(H)$.

Assuming the value of the local Reynolds number at the point of stability loss $Re_{\bar{x}_1} = 1{\cdot}4 \,.\, 10^5$ we obtain the relation $H(Re_{\bar{x}})$ which is plotted in Fig. 2. In the same figure the relation $Re^{**}(Re_{\bar{x}})$ is plotted using equation (3).

From equation (6) for a special case of a plate with suction we have

$$\bar{v}_0 = \frac{v_0}{u_0} = \frac{C_3}{Re^{**}}. \qquad (8)$$

Using $C_3(H)$, $Re^{**}(H)$ and $H(Re_{\bar{x}})$ the expression $\bar{v}_0 = f(Re_{\bar{x}})$ is obtained. The relation between the local skin-friction factor and the local $Re_{\bar{x}}$ may be found by the formula

$$C_f = \frac{2\zeta(H)}{Re^{**}(H)}. \qquad (9)$$

Further calculation of the boundary layer on bodies of revolution with distributed suction is not difficult, and as was demonstrated by calculations for bodies of revolution, only one approximation is required.

5.

Estimation of the optimum suction rate along a body according to formula (6) is simplified by the plots which are applicable to any pressure gradient. For this purpose the form parameter $f = \delta^{**2}\bar{u}'Re$ is introduced.

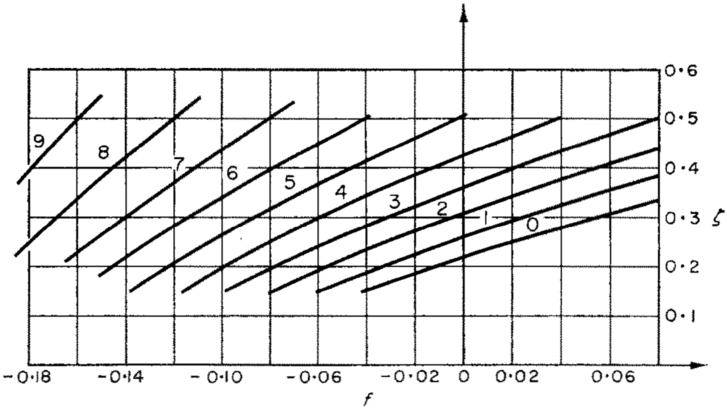

FIG. 3. Function ζ versus form parameter and suction intensity

0. $t^{**} = 0{\cdot}0$	5. $t^{**} = 0{\cdot}5$
1. $t^{**} = 0{\cdot}1$	6. $t^{**} = 0{\cdot}6$
2. $t^{**} = 0{\cdot}2$	7. $t^{**} = 0{\cdot}6$
3. $t^{**} = 0{\cdot}3$	8. $t^{**} = 0{\cdot}8$
4. $t^{**} = 0{\cdot}4$	9. $t^{**} = 0{\cdot}9$

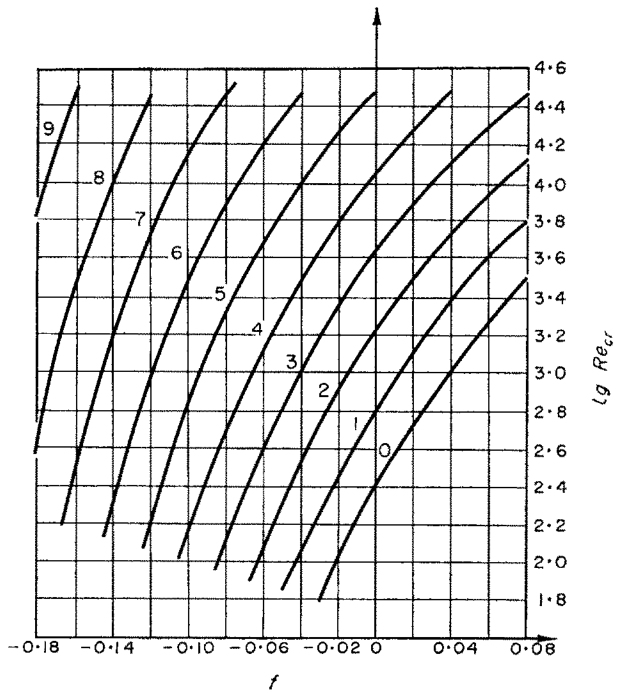

FIG. 4. Critical Reynolds number Re_{cr}^{**} versus form parameter and suction intensity

0. $t^{**} = 0{\cdot}0$	5. $t^{**} = 0{\cdot}5$
1. $t^{**} = 0{\cdot}1$	6. $t^{**} = 0{\cdot}6$
2. $t^{**} = 0{\cdot}2$	7. $t^{**} = 0{\cdot}6$
3. $t^{**} = 0{\cdot}3$	8. $t^{**} = 0{\cdot}8$
4. $t^{**} = 0{\cdot}4$	9. $t^{**} = 0{\cdot}9$

Equation (6) yields

$$\bar{v}_o\delta^{**}Re = C_1 f + C_3. \tag{10}$$

Relating the local suction rate to the flow velocity at the extremity of the boundary layer $\bar{u} = u/u_o$ we get

$$\frac{\bar{v}_o}{\bar{u}} = \frac{C_1}{Re^{**}} f + \frac{C_3}{Re^{**}}. \tag{11}$$

After introduction of the suction intensity

$$t^{**} = -\bar{v}_o\delta^{**}Re \tag{12}$$

from equation (10)

$$-t^{**} = C_1 f + C_3. \tag{13}$$

For $C_1(H)$ and $C_3(H)$ it follows from Fig. 1

$$C_1 = -3{\cdot}48 + 3H \tag{14}$$

$$C_3 = -2{\cdot}16 + 0{\cdot}83H. \tag{15}$$

Solving the system of equations (13), (14) and (15) with respect to H we obtain

$$H = \frac{2{\cdot}16 - t^{**} + 3{\cdot}48f}{3f + 0{\cdot}83}. \tag{16}$$

Using $\zeta(H)$ and $Re_{cr}^{**}(H)$ the relations $\zeta(f, t^{**})$ and $Re_{cr}^{**}(f, t^{**})$ are plotted in Figs. 3 and 4.

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Abstract—A method is presented for laminar boundary-layer calculation with optimum distribution of suction. The relation $H(\bar{x})$ used as a first approximation enables graphs to be drawn (Figs. 2–4) with calculations of the local skin friction and the critical Reynolds number Re_{cr}^{**} along the length of a body considerably.

The optimum suction rate and the parameter H are determined according to formulae (11) and (16) respectively.

Résumé—Cet article présente une méthode de calcul de couche limite laminaire avec distribution optimale d'aspiration. La relation $H(\bar{x})$ utilisée comme première approximation a permis de tracer les courbes des figures 2 à 4 qui facilitent considérablement les calculs du frottement local et du nombre de Reynolds critique au long d'un corps.

Le taux d'aspiration optimal et le paramètre H sont respectivement déterminés par les formules (11) et (16).

Zusammenfassung—Zur Berechnung der laminaren Grenzschicht mit optimaler Absaugungsverteilung ist eine Methode angegeben. Die als erste Näherung herangezogene Beziehung $H(\bar{x})$ gestattet Diagramme (Fig. 2–4) au zeichnen, welche die Berechnung der örtlichen Oberflächenreibung und der kritischen Reynoldszahl Re_{cr}^{**} längs der Körper beträchtlich erleichtern. Die optimale Sauggeschwindigkeit und der Parameter H wird nach Gleichung (11) bzw. (16) bestimmt.